375. Proposed by S. LEFSCHETZ, University of Nebraska.

Prove that $\frac{e}{2m+2}<e-(1+\frac{1}{m})^m<\frac{e}{2m+1}$. [Schlömilch.]

Solution by H. E. TREFETHEN, Colby College.

Dividing by e and subtracting each member from unity, we deduce

$$1/(1+1/2m)<(1+1/m)^m/e<(1+1/2m)/(1+1/m)\ldots(1).$$

(i) Let n be positive, and put $m=-n$. Then (1) becomes

$$1/(1-1/2n)<(1-1/n)^{-n}/e<(1-1/2n)/(1-1/n)\ldots(2).$$

$-\log(1-1/2n)<-n\log(1-1/n)-1<\log(1-1/2n)-\log(1-1/n)$ or

$$1/2n+1/2^2.2n^2+1/2^3.3n^3+\ldots+1/2^r rn^r$$
$$<1/2n+1/3n^2+1/4n^3+\ldots+1/(r+1)n^r$$
$$<1/2n+(2^2-1)/2^2.2n^2+(2^3-1)/2^3.3n^3+\ldots+(2^r-1)/2^r rn^r.$$

These series all converge when $n>1$. We also have

$1/2^r rn^r<1/(r+1)n^r<(2^r-1)/2^r rn^r$.

$2^r>(r+1)/r>2^r/(2^r-1$, $2^r-1>1/r>1/(2^r-1)$.

For $2^r-1>r>1/r$ when $r\mathrel{\overline{\overline{>}}}2$ (r being a positive integer).

Thus the given relations are established for the case when $n>1$, $-n<-1$, that is when $m<-1$.

(ii) In (1) put $m=n-1$. Then after multiplying by $n/(n-1)$ we find

$$1/(1-1/2n)<(1-1/n)^{-n}/e<(1-1/2n)/(1-1/n).$$

This is the same as (2) in (i) and the case is proved when $n>1$, $n-1>0$, that is when $m>0$.

(iii) If $m=0$, the given expressions become $e/2<e-1<e$ and the case is proved for $m=0$.

(iv) But in the interval $0>m\mathrel{\overline{\overline{>}}}-1$ there are various results for different values of m.

(a) For such values of m as render the given functions real and finite, the given inequalities are true if $0>m>1/2$, but must be reversed if $-1/2>m>-1$.

(b) If $m=-1/2, -1/4, -3/4, \ldots, -\frac{[2p-1]_{p=1}^{p=q}}{2q}$, $(1+\frac{1}{m})^m$ is imaginary.

(c) If $m=-1/2$, $e<e+\sqrt{(-1)}<\infty$.

(d) If $m=-1$, $\infty<e-\infty<-e$.

Therefore the given statements are proved true unless $0>m\overline{\gtrless}-1$. For case (iv) no general statement of relative magnitudes can be made on account of discontinuous functions.

376. Proposed by W. W. BEMAN, Professor of Mathematics, University of Michigan, Ann Arbor, Michigan.

If $\frac{(1+1/m)^m}{e}=1-a_1\frac{1}{m}+a_2\frac{1}{m^2}-a_3\frac{1}{m^3}+\ldots$, prove $na_n=\sum_{k=1}^{k=n}\frac{k}{k+1}a_{n-k}$, and compute a_1, a_2, a_3, ..., a_8.

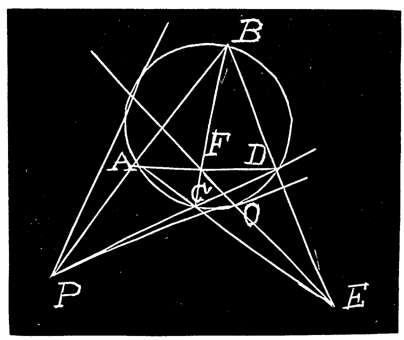
No solution of this problem has been received.

GEOMETRY.

400. Proposed by FRANCIS RUST, C. E., Pittsburgh, Pennsylvania.

Given a circle and a point P without; construct, *using the straight edge only*, the two tangents to the circle through P.

II. Solution by GEORGE W. HARTWELL, Hamline University, St. Paul, Minnesota.



Through P draw any two secants AB and CD, cutting the circle in A, B, C, and D. Join A and D, and A and C; B and D, and B and C. AC and BD meet at E, and AD and BC meet at F. Join E and F. EF is the polar of P. Then the points O and M in which the line EF intersects the circle are the points of tangency.

402. Proposed by H. PRIME, Boston, Mass.

The diameter of a hoop-shaped ring (or collar) is 24 inches at one edge and 28 inches at the other edge. A cross-section is a crescent with circular arcs of 120° and 60°, whose common chord is 4 inches long. Find its volume by elementary methods (without the use of calculus or the center of gravity).

Solution by H. E. TREFETHEN, Colby College.

Denote the given chord by AB, the axis of the ring by QQ, the arc of 120° by s, of 60′ by s'. Let ABC be an equilateral triangle. Complete the arcs s and s', and through A and C draw their diameters parallel to QQ.